Mitigating destabilization effects due to line dynamics in multi-inverter systems

Guru Praanesh Raman

NUS

Outline

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Why generalized droop?

Conventional droop equation:

$$\begin{bmatrix} \Delta f \\ \Delta V \end{bmatrix} = - \begin{bmatrix} k_f & 0 \\ 0 & k_v \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{1}$$

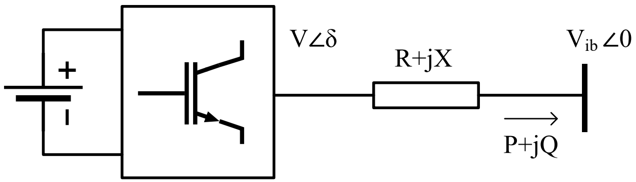

Figure 1: Inverter connected to an infinite bus through $Z\angle\theta = R + jX$.

General power flow equations

$$P = \frac{VV_{ib}}{Z}(\sin\theta)\ \delta + \frac{V_{ib}}{Z}(\cos\theta)\,(V - V_{ib}) \tag{2}$$

$$Q = -\frac{VV_{ib}}{Z}(\cos\theta)\ \delta + \frac{V_{ib}}{Z}(\sin\theta)\,(V - V_{ib}) \tag{3}$$

The generalized droop law

Generalized droop equation:

$$\begin{bmatrix} \Delta f \\ \Delta V \end{bmatrix} = - \begin{bmatrix} k_f \sin\theta & -k_f \cos\theta \\ k_v \cos\theta & k_v \sin\theta \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{4}$$

General power flow equations

$$P = \frac{VV_{ib}}{Z}(\sin\theta)\ \delta + \frac{V_{ib}}{Z}(\cos\theta)\,(V - V_{ib}) \tag{5}$$

$$Q = -\frac{VV_{ib}}{Z}(\cos\theta)\ \delta + \frac{V_{ib}}{Z}(\sin\theta)\,(V - V_{ib}) \tag{6}$$

The droop occurs in a rotated power frame $[\Delta\hat{P}, \Delta\hat{Q}] = \mathbf{T}\,[\Delta P, \Delta Q]$, with:

$$\mathbf{T} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \otimes \mathbf{I_n} \text{ and } \mathbf{T} \begin{bmatrix} \mathbf{B} & -\mathbf{G} \\ \mathbf{G} & \mathbf{B} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{B}} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{B}} \end{bmatrix}$$

Here, $\phi = 0.5\pi - \theta$ and $\mathbf{I_n}$ is the identity matrix.

Control Schematic

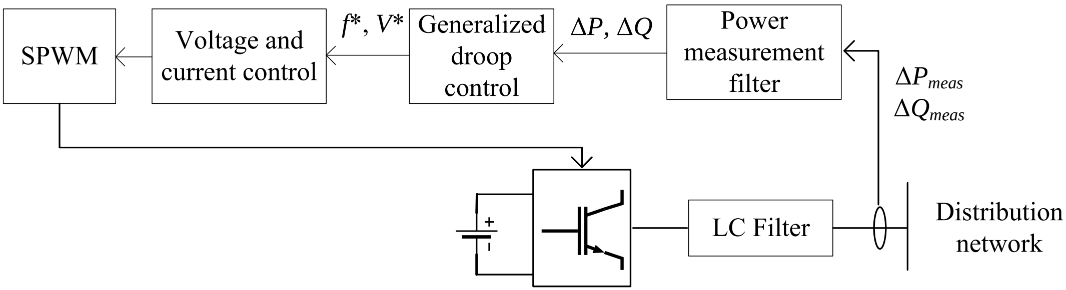

Figure 2: Control schematic of an inverter with generalized droop.

Analysis approach

- First, **use generalized droop** to remove instability mechanism due to cross-coupling
- Incorporate EM dynamics to **observe remaining instability factors**
- **Develop control modification(s)** to achieve guaranteed dynamics stability for all droops and R/X values

Third-order model

The characteristic equation with static Jacobian is:

$$\left(\begin{bmatrix} s(T_c s+1)\mathbf{\Lambda_f} & \mathbf{0} \\ \mathbf{0} & (T_c s+1)\mathbf{\Lambda_v} \end{bmatrix} - \mathbf{T}\begin{bmatrix} \mathbf{B} & -\mathbf{G} \\ \mathbf{G} & \mathbf{B} \end{bmatrix}\right)\begin{bmatrix} \mathbf{\Delta}\boldsymbol{\delta} \\ \mathbf{\Delta}\boldsymbol{V} \end{bmatrix} = \mathbf{0} \tag{7}$$

where $\mathbf{\Lambda_f} = \mathbf{K_f}^{-1}$ and $\mathbf{\Lambda_v} = \mathbf{K_v}^{-1}$.

<u>Illustration</u>- the single-inverter case:

$$T_c s^2 + s - k_f \hat{B} = 0 \tag{8}$$

$$T_c s + 1 - k_v \hat{B} = 0 \tag{9}$$

The oscillatory poles are **always stable**:

$$s = -\frac{1}{2T_c} \pm \frac{1}{2T_c}\sqrt{1 + 4k_f \hat{B} T_c} \tag{10}$$

The real pole $s = -(1 - k_v \hat{B})/T_c$ is **always stable** as well.

Incorporating line dynamics

We now use dynamic power system model as:

$$\begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} = - \begin{bmatrix} \mathbf{B}(s) & -\mathbf{G}(s) \\ \mathbf{G}(s) & \mathbf{B}(s) \end{bmatrix} \begin{bmatrix} \mathbf{\Delta \delta} \\ \mathbf{\Delta V} \end{bmatrix}, \tag{11}$$

with the substitutions:

$$\begin{aligned} \mathbf{G}(\mathbf{s}) &= -\frac{(\rho^2+1)(\rho+s/\omega_0)}{(\rho+s/\omega_0)^2+1}\mathbf{B} \\ \mathbf{B}(\mathbf{s}) &= \frac{(\rho^2+1)}{(\rho+s/\omega_0)^2+1}\mathbf{B} \end{aligned} \tag{12}$$

Here, ρ denotes the R/X ratio of the lines and ω_0, the power frequency 100π rad/s.

Fifth-order model

With EM dynamics, the model may be obtained as:

$$\left(\underbrace{\frac{(\rho+s/\omega_0)^2+1}{(\rho^2+1)}}_{DSLF}\begin{bmatrix} s(T_c s+1)\mathbf{\Lambda_f} & \mathbf{0} \\ \mathbf{0} & (T_c s+1)\mathbf{\Lambda_v}\end{bmatrix} - \begin{bmatrix}\hat{\mathbf{B}} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{B}}\end{bmatrix} + \underbrace{\frac{s}{\omega_0}\begin{bmatrix} -\mathbf{B}\sin\phi & -\mathbf{B}\cos\phi \\ \mathbf{B}\cos\phi & -\mathbf{B}\sin\phi\end{bmatrix}}_{\text{non-decoupled}}\right)\begin{bmatrix}\mathbf{\Delta\delta} \\ \mathbf{\Delta V}\end{bmatrix} = \mathbf{0} \qquad (13)$$

Phenomena to analyze:

- Distribution System Lag Factor (DSLF)
- EM-induced cross-coupling ($\cos\phi$ terms)
- EM-induced damping ($\sin\phi$ terms)

Distribution System Lag Factor (DSLF)

Temporarily neglecting cross-coupling term ($\cos\phi$ term),

$$\left(\frac{s^2}{\omega_0^2} + \frac{2\rho}{\omega_0}s + 1 + \rho^2\right) s(T_c s + 1) - (1+\rho^2)k_f\hat{B} - \frac{(1+\rho^2)B\sin\phi}{\omega_0} = 0$$

Using Routh-Hurwitz method, the stability condition is:

$$k_f < \frac{\omega_0(1+\rho^2)}{B(1+\rho^2)\sin\phi - 2\rho\hat{B}} \tag{14}$$

$\implies$ Stability is lost at higher k_f!

- Unity gain at zero frequency $\implies$ Does not change steady-state droop
- High lag at low R/X (1-3) and lower lag at higher R/X (>5)
- At high R/X, DSLF $\rightarrow$ 1

Conclusion: DSLF destabilizes system at lower R/X and has no effect at higher R/X

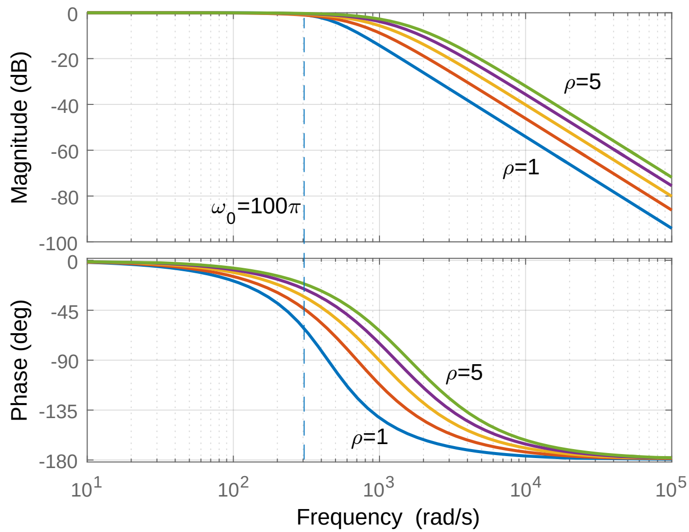

Figure 3: Bode plot of $1/DSLF$ for various R/X ratios indicating significant phase lag at power frequency ω_0.

EM-induced damping

Taking $DSLF = 1$, we obtain the characteristic equations as:

$$T_c s^2 + \left(1 - \frac{k_f B \sin\phi}{\omega_0}\right) s - k_f \hat{B} = 0 \tag{15}$$

$$\left(T_c - \frac{k_v B \sin\phi}{\omega_0}\right) s + 1 - k_v \hat{B} = 0 \tag{16}$$

- Since B and $\hat{B}$ are both negative, system is always stable
- The $\sin\phi$ terms increase damping, pushing poles to the left of $-\omega_c/2$

EM-induced cross-coupling

- Cross-coupling appears due to $\cos\phi$ terms
- This effect pushes poles rightward, but does not affect stability:

$$\left(T_c + \frac{k_f k_v B^2 \cos^2\phi}{\omega_0^2(1-k_v\hat{B})}\right)s^2 + \left(1 - \frac{k_f B \sin\phi}{\omega_0}\right)s - k_f\hat{B} = 0 \quad (17)$$

- Cross-coupling disappears at high R/X ratios as $\cos\phi \to 0$

EM effects summary

- DSLF is the key destabilization factor
- EM-induced damping improves damping
- EM-induced cross-coupling slightly reduces damping, but does not affect stability
- **Stability is guaranteed iff effect of DSLF is annulled, for all droops and R/X ratios**

Proposed power filter

The initial design of the filter was:

$$F_0(s) = \frac{1}{T_c s + 1} \tag{18}$$

The proposed filter is:

$$F(s) = \frac{s^2/\omega_0^2 + 2\rho s/\omega_0 + 1 + \rho^2}{(\rho^2+1)(T_c s+1)(\tau s+1)} \tag{19}$$

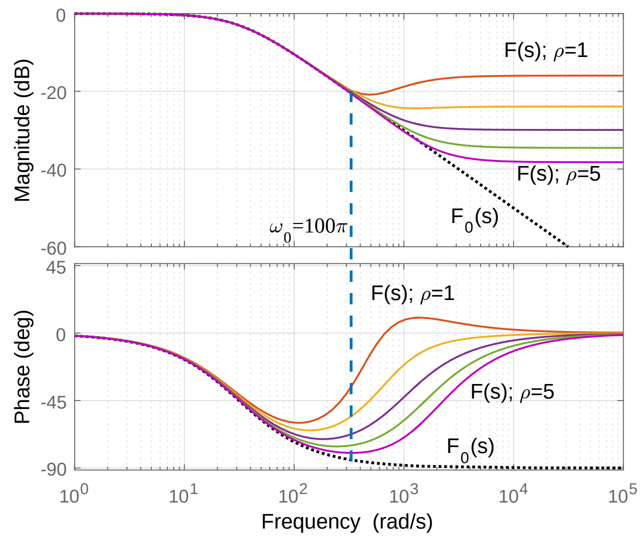

Figure 4: Bode plots for original filter $F_0(s)$ and the proposed filter $F(s)$ for various values of ρ. The proposed filter provides phase lead at $\omega_0 = 314$ rad/s.

Selection of τ

- The dummy term $(\tau s + 1)$ makes $F(s)$ causal. We need $\tau << T_c$.
- The phase lead at $\omega_0 = 314$ rad/s is higher for lower values of τ.
- Higher values of τ require high BW filter design. This is however unnecessary as τ=1e-3 is quite sufficient.

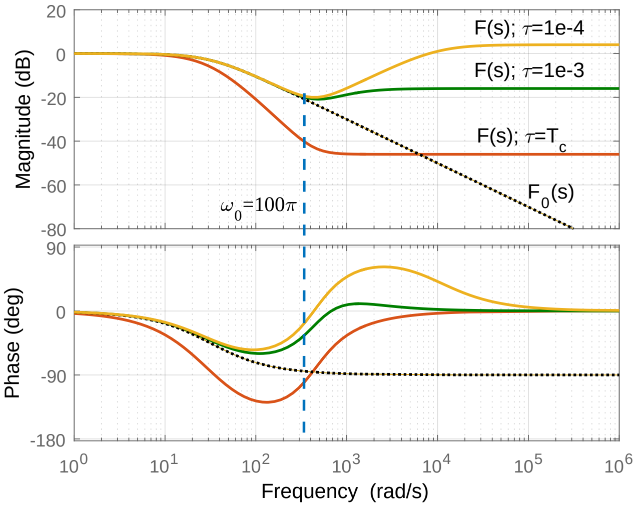

Figure 5: Bode plots for original filter $F_0(s)$ and the proposed filter $F(s)$ for various values of τ.

Closed loop poles

- The implementation of the proposed filter eliminates the effect of the DSLF
- The eigenvalues lie to the left of the vertical line at $-\omega_c/2$ rad/s.
- The poles do not lie exactly on that vertical line because of the EM-induced damping effect.

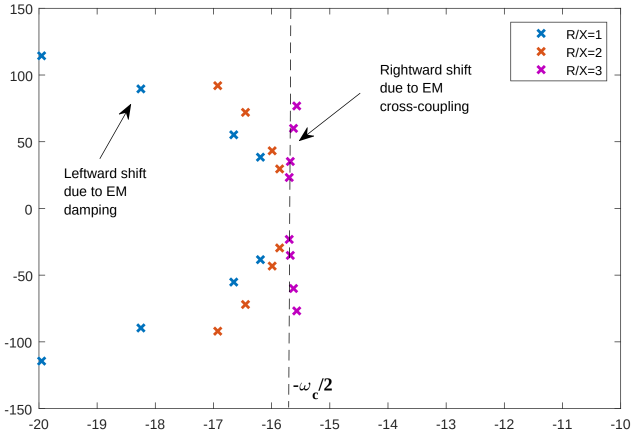

Figure 6: Closed loop poles for 5-inverter system illustrating discussed effects

Time-domain results

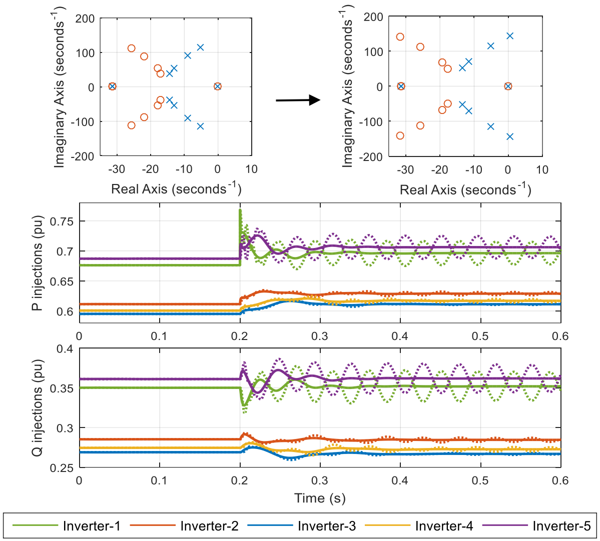

Figure 7: *Top*: Initial and final locations of poles for conventional filter (blue x) and proposed filter (red o). *Below*: Evolution of real and reactive power injections with the conventional filter (dashed lines) and proposed filter (solid lines) for the same droop settings, indicating well-damped behavior with the proposed filter.

Stability region

- With the conventional filter, the stability region is smaller for smaller R/X values, and larger for larger R/X values.
- With the implementation of the proposed filter, the stability region becomes infinite.
- This reiterates that the DSLF is the sole instability factor to be addressed.

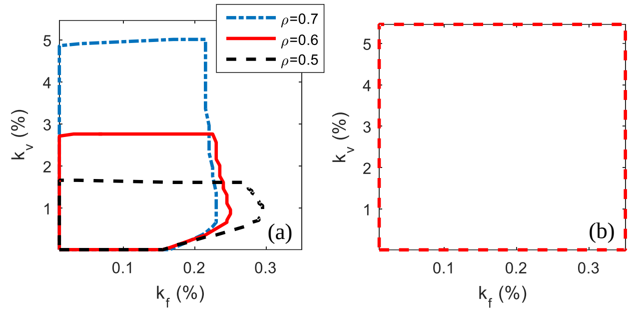

Figure 8: Stability region for generalized droop under various R/X ratios with (a) conventional filter (b) proposed filter (the entire plane is theoretically stable for (b)).

Application to conventional droop

- Under conventional droop, both the instability phenomena: P-V/Q-f cross-coupling, and line dynamics exist.
- With the proposed filter, the stability region significantly expands for lower R/X values, and slightly for higher R/X.
- This implies that cross-coupling and line dynamics are *independent* instability phenomena, the former dominant at higher R/X values, and the latter at lower R/X.

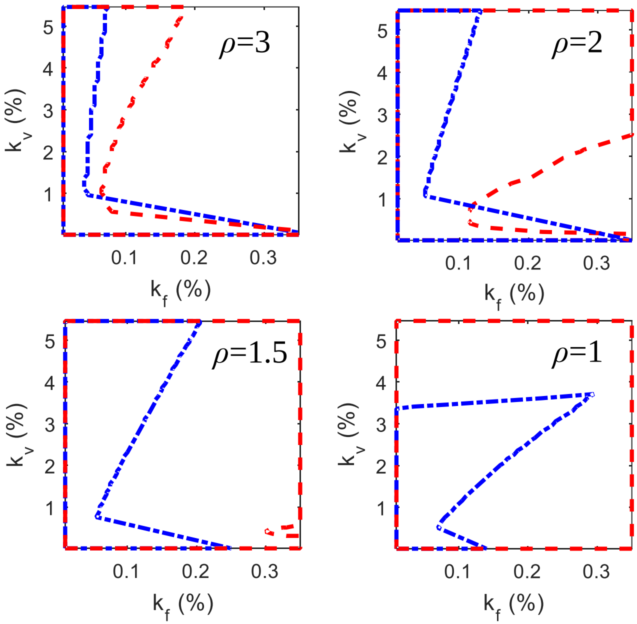

Figure 9: Stability region comparison for P-f/Q-V droop under various R/X ratios with conventional filter (blue dash-dotted lines) and proposed filter (red dashed lines) indicating

Conclusions

- Line dynamics is an important stability in practical distribution systems with moderate or low R/X ratios (< 3)
- The proposed filter has been verified to fully eliminate this instability effect
- The proposed filter is also valid for systems with conventional droop control
- The design procedure is formula-based, independent of the system topology and operating point, and does not require any hardware modifications.